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FINAL REPORT

Project Title: Adaptive Lens Inspired by Bio-Visual Systems

Award No. F49620-02-1-0426

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1. Projective Objectives

The objectives of our BOSS I research are

- (a) to identify and demonstrate designs and technologies that can meet the performance requirements for bio-inspired adaptive lenses,
- (b) to fabricate and evaluate device prototypes,
- (c) to identify potential issues and technological challenges that may become road blocks for real military and civilian applications,
- (d) to advance the science and technology of bio-inspired adaptive optics which has the potential to create a paradigm shift in the field of optics.

2. Summary of the Accomplishments

During the DARPA BOSS I program, the UCSD team has made substantial progress in the following areas:

- (a) We have identified and demonstrated the merits of PDMS elastomer for lens membranes. The PDMS-based fluidic lens process has been proven to be simple, controllable, and scalable to form lenses from 10 μ m to several centimeters in diameter.
- (b) We have demonstrated fluidic tunable lenses with an exceedingly wide tuning range and strong lens power. Most tunable lenses that have been demonstrated today can only achieve a large f-number (or weak lens power) in the order of 10 to 15. Our tunable lens is the first and only device that can achieve an f-number of 0.7 and an extremely wide tuning range from f# = 0.7 to >100. The strong lens power and the wide tuning range are the keys to many superior and unique functions of imaging systems.
- (c) We have successfully extended the single-chamber fluidic lens structure into multichamber lens structures. As a result, one can not only achieve a wide tuning range in focal distance, f-number, and numerical aperture, but also obtain the extreme flexibility in the form of lens types. We have demonstrated that the same lens structure can be transformed into any of the following structures: plano-convex, plano-concave, double-convex, double-concave, positive-meniscus, and negativemeniscus, reversibly.

- (d) Following the same course of development, we also demonstrated, for the first time, an integrated zoom lens without mechanical moving parts. The device consists of two back-to-back tunable lenses separated by a spacer of a few millimeters in thickness. The two lenses and the spacer substrate are all bonded together during the wafer process, which suggests that the zoom lens can be fabricated using standard microfabrication process like integrated circuits. To achieve the zooming function, the focal distances of both lenses are adjusted. There is no need for changing the distance between the lenses, leading to a more compact design and a structure suitable for integration and miniaturization.
- (e) Due to the combined features of strong lens power, lens type convertibility, and extremely wide tunability, we have demonstrated the unique feature of combining a telephoto system and a reverse telephoto (also known as wide-angle) system in one. This leads to a continuous tuning angle of field-of-view from 7 degrees to 65 degrees.

3. Detailed Technical Discussion

3.1. Basic fluidic lens process and characteristics

Figure 1 shows the basic process of forming fluidic adaptive lenses. We used micromolding technique and plasma-assisted bonding process to form lens chamber and lens membrane. A Si mold master was formed using standard microfabrication and MEMS process. After surface treatment of the Si mold to facilitate demolding, PDMS was deposited to the Si mold. The body of the lens chamber was formed after separating the PDMS material from the Si mold master. To form the lens membrane, another layer of PDMS film was deposited to a Si handle wafer. After oxygen plasma activation of the PDMS surfaces, the PDMS membrane was covalently bonded to the PDMS lens chamber. The lens body was formed after removing the bonded PDMS structure from the Si handle wafer, as shown in Fig. 1(d). The thickness of the PDMS membrane is typically 30 to 100 μm . The bonded PDMS structure has good quality and uniformity. No fluidic leakage or film delamination was observed for the bonded structure.

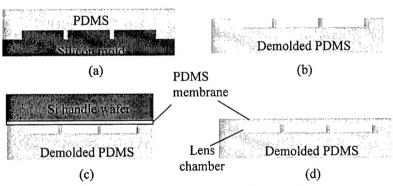


Figure 1. Process flow for tunable fluidic lenses.

Figure 2 shows the basic focal length tuning characteristics of fluidic lenses with two different media: DI water (n=1.33) and sodium chromate (n=1.50). The lens aperture is 20 mm and the shortest focal distance achieved in this design is 14mm with sodium chromate as the medium. Experiment on lenses of similar design but with different dimensions establishes the rule of thumb that the shortest focal distance is about 70% of the lens aperture. This is, to our knowledge, the widest focal distance tuning range and the shortest focal distance, ever been demonstrated for any types of adaptive lenses. The data of numerical aperture and field of view are shown in Figures 3,4. The numbers are close to the characteristics of fish eyes.

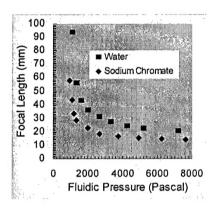


Figure 2. Dependence of focal distance on the fluidic pressure for a 20 mm diameter adaptive plano-convex lens. The data show the measured results for two different lens media, water (n=1.33) and sodium chromate (n=1.55).

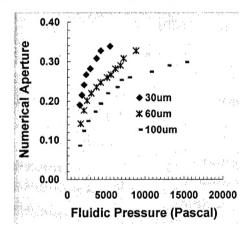


Figure 3. Tuning characteristics of numerical aperture (NA) with fluidic pressure for lenses having different membrane thickness.

Besides wide tunability, the lenses need to produce high quality image. Figures 4,5 show the measurements of image resolution using the USAF standard. The measurement was performed at a fixed image distance of 635 mm using a He/Ne laser as the light source. In all lens structures we have investigated, the resolution is greater than 40 line pairs per millimeter (lp/mm), which is comparable to solid-state lenses made of glass and quartz and significantly better than molded plastic lenses. The values of resolution were chosen at a modulation transfer function (MTF) value of 0.5.

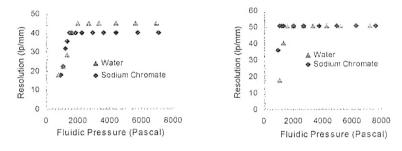


Figure 4. Image resolution for plano-convex (a) and biconvex (b) lenses.

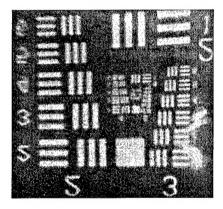


Figure 5. Picture of resolution measurement using negative USAF standard on a 20 mm fluidic lens at 43 mm focal distance.

The rapid reduction of the image resolution at low fluidic pressure (or long focal distance) was partly attributed to the reduction of the magnification factor and the resolution limit of the CCD camera. However, we should also point out that part of the reduced resolution could be due to the asymmetric lens shape caused by the gravity of he lens medium. This speculation was supported by the findings that the resolution reduction effect tends to be more pronounced for lenses of larger aperture size and thinner membrane. One way to minimize the effect would be increasing the membrane

thickness with the aperture of the lens. Another way, which is more effective although at the expense of the power consumption of the fluid pump, is to introduce a built-in tensile stress in the elastomer membrane. When the built-in tensile stress is significantly greater than the pressure due to the gravity of the fluid, the shape distortion caused by the gravity effect is negligible.

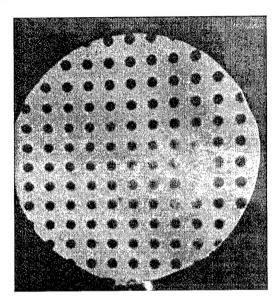


Figure 6. Photograph for image distortion measurement on a 60 μ m membrane adaptive lens. The dot diameter is 1.00 mm and the center-to-center spacing between dots is 2.00 mm in the grid distortion target.

Figure 6 shows the measurement of image distortion caused by spherical aberration. The standard target pattern has a dot diameter of 1 mm and a dot-to-dot spacing of 2 mm. The photograph was taken when the lens was tuned to a focal distance of 40 mm. Similar image quality was obtained over a wide range of focal distance, suggesting that the image distortion does not impose serious problems on image quality. The photograph (Fig. 6) was achieved with a lens of a uniform membrane thickness. It was found that by using a membrane of specially designed thickness profile, the spherical aberration of the lens can be reduced. However, the control of the thickness profile is technologically challenging and the optimal thickness profile changes with lens aperture and lens medium, making the approach hard to apply in real optical systems. Since one can always use aspherical lenses of fixed properties to eliminate most of spherical aberration, we intend to use a uniform lens membrane to construct optical systems for most of our research.

3.2. Lens type versatility

We have extended the design of simple lenses to more complicated lens structures. One of such examples is the lens with 2 fluidic chambers with valves so the chamber pressure can be controlled separately. A typical structure of such double-chamber lens and the fluidic and electronic control circuits are shown in Fig. 7. The most salient feature of such double-chamber lens is that not only the lens properties but also the lens type can be tuned. By controlling the pressure of each lens chamber, the value of which can be positive or negative relative to the ambient, one can achieve any of the following lens types: plano-convex, plano-concave, biconvex, biconcave, positive meniscus, negative meniscus. Figure 8 shows one example how different lens types can be obtained via control of the pressure of the lens chambers.

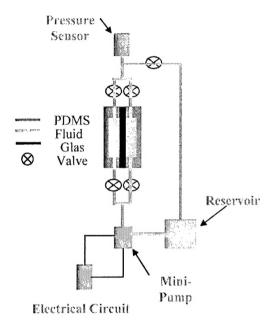


Figure 7. Diagram of the double-chamber fluidic lens and the peripheral control circuit. Depending on the sign and magnitude of the pressure in each chamber, the same lens can be made into different lens types.

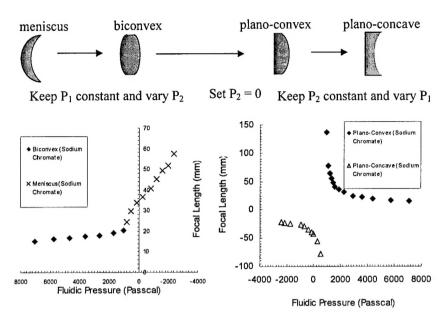


Figure 8. Demonstration of lens type convertibility among biconvex, meniscus, planoconvex, and plano-concave lenses.

Such extreme versatility is beyond what has been demonstrated by any other types of adaptive lenses and may have significant impact on optics industry. For one thing, one can use one standard double-chamber fluidic lens to replace myriad types of lenses fabricated for specific applications, thus cutting down the inventory and production cost tremendously. For another thing, one can functionally integrate optic systems of very different or even opposite functions into one system. We will discuss the latter point in more detail next.

3.3. Integrated single-chip zoom lens

Zoom lenses are perhaps the most desirable but technologically challenging devices for any miniature imaging systems. The ability of fast zooming and high zoom ratio is critical to target recognition, motion detection, and tracking. A typical imaging system needs a zoom ratio of 3-6, and some special systems may require a zoom ratio greater than 10 or even as high as 30. In current systems, zooming is achieved by varying the distance between two lenses. An apparent drawback of such design is the requirement of large range (several millimeters or centimeters) mechanical motion, which makes zoom lens systems clumsy, expensive, and difficult to integrate into an array form. Here we devised a different zoom lens using the tunable properties of the lenses. The zoom lens consists of two back-to-back tunable fluidic lenses, separated by a substrate spacer. The lenses and the substrate spacer are bonded together using the plasma-activated bonding

process, so the process is done in wafer scale and zoom lens arrays can be fabricated. Analysis shows that the zoom ratio can be represented as a function of image distance, object distance, focal distances of the front (object side) and back (image side) lenses, and the lens spacing. For all practical applications, we can assume that the object distance approaches infinity. Then the changeable parameters are image distance, lens spacing, and the focal lengths. Instead of physically changing the image distance and lens spacing as people do with conventional zoom lenses, we can achieve the same zooming effect by varying the focal distances of the front and the back lenses. The zoom ratio can be approximated as

$$ZR = 1 + d/i + d/|fm| \tag{1}$$

The second term in Eq. (1) is the ratio of the lens spacing, d, and the image distance, i, defined as the physical distance between the image plane and the plane of the rear lens. The third term in Eq. (1) is the additional contribution from the lens type versatility, where |fim| is the minimal magnitude of the focal distance the lens can achieve as a concave lens. Without the flexibility in lens type, the third term vanishes and the achievable zoom ratio is reduced. For a design of 5 mm image distance, 6 mm lens spacing and a minimal magnitude of the focal distance of 4 mm, we can achieve a zoom ratio of 3.7, which is compared favorably to optical zoom lenses in digital cameras. Should high index fluid is used to replace DI water as the lens medium, a zoom ratio greater than 5 can be readily achieved. The detailed analysis of the design and projected performance of fluidic zoom lens is shown in the Appendix.





Figure 9. Demonstration of the zooming effect of an integrated fluidic zoom lens. Zooming was achieved without changing the spacing between lenses.

Figure 9 shows the photograph of the test setup for "integrated fluidic zoom lens". The zoom lens is mounted on a holder and connected to the external fluidic control circuit. The fluidic control circuit, including valves, pumps, and sensors, can be integrated with the zoom lens into a compact module although they are discrete components in the

current setup. Figure 10 shows an example of the zooming function for the integrated zoom lens. Due to a more conservative design, a zoom ratio of 1.5 was demonstrated on this device. An improved lens design using similar fabrication process produces a much higher zoom ratio, as shown in Figure 11 and Table 1.

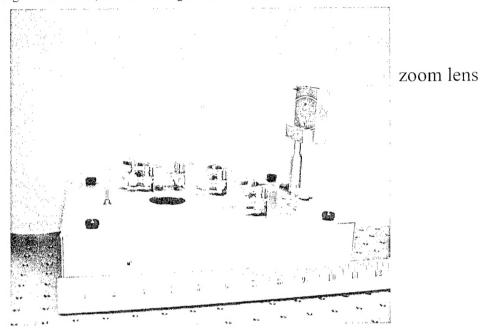


Figure 10. Photograph of the test setup for integrated fluidic zoom lens.

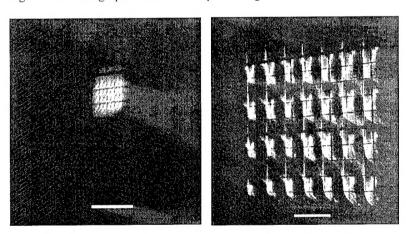


Figure 11. Image of an 4x7 LED array obtained by a tunable fluidic zoom lens. The yellow bars at the bottom of each photo are of the same length, so the size of the image represents the (de)magnification achieved by zooming.

Object (mm)	Image (mm)	Zoom Ratio
250	50	3.05
250	30	4.18
1000	50	3.58
1000	30	4.44

Table 1. Zoom ratios demonstrated by an integrated fluidic zoom lens at different image distances.

3.4 Integration of telephoto and reverse telephoto system: tunable FoV

Besides zooming, tunable field-of-view is another important feature for surveillance cameras. Achieving a wide range of field of view has been one of the main project goals for DARPA BOSS initiative, as depicted in the original BAA. The UCSD team has tried to develop a zoom lens system with a widest tuning range in field-of-view. As a result of technology and design optimization, we have demonstrated a zoom lens system with a tunable field-of-view from 7-degrees to 65-degrees.

To formally characterize the field-of-view, we use the established criteria in photography for telephoto systems and reverse telephoto (wide-angle) systems. The definition of a telephoto system is that its "effective focal length (EFL)" is greater than the "overall physical length (OAL)", measured from the first surface of the lens to the image plane (i.e. EFL/OAL >1). On the other hand, the definition of a reverse telephoto (wide angle) system is that the "back focal length (BFL)" is larger than the "effective focal length (EFL)" or BFL/EFL > 1. Under these formal criteria, many of the so-called telephoto cameras in the commercial market are not really telephoto systems. Furthermore, to our best knowledge, there are no commercial cameras that can work both as a telephoto camera and as a wide-angle (fish-eye) camera without physically replacing the lens set. In contrast, figure 12 shows that, because of the extremely wide tuning range and lens type convertibility, the zoom lens system made of our tunable lenses can function as a telephoto system and as a reverse telephoto system using the same set of optics, a feat that has never been demonstrated before!

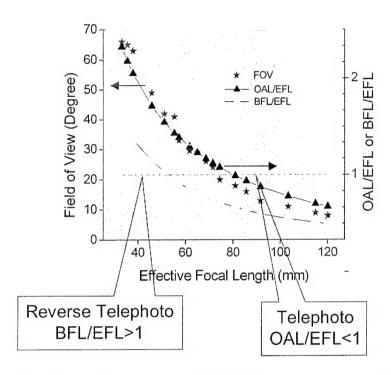


Figure 12. Demonstration of a tunable zoom lens with a wide field-of-view (from 7 degrees to 65 degrees) that can be used as a telephoto system and a reverse telephoto (wide-angle) system.

As indicated by the yellow dots in Fig. 12, when the effective focal length becomes shorter than 55mm (for 20mm aperture), the condition of BFL/EFL > 1 is satisfied and the system functions as a true reverse telephoto system. At the minimal EFL of 20 mm, the BFL/EFL ratio rises to 1.5 and the widest field-of-view of 65 degrees (as indicated in the "red stars" in Fig. 12) is achieved. Following the trace of the "black triangles", we find that OAL/EFL < 1 when the effective focal length is longer than 80mm, making the system a true telephoto system. At the longest achievable effective focal distance of 120 mm, a minimal ratio of 0.5 is achieved, corresponding to a field-of-view of 7 degrees. Therefore, we have demonstrated that the tunable lenses allow us to construct a system that can be continuously tuned between a telephoto system and a reverse telephoto system.

4. Discussions

Our research in the BOSS I program has been extremely productive and exciting. As summarized in the previous sessions, many ground breaking results and technology breakthroughs were made. As a result of our research, the state of the art of tunable

lenses has been advanced significantly. On the other hand, equally important to the technical community are many lesions and issues learned in the course of our research endeavor. These lessons and issues, as to be discussed next, provide insight and direction for our future research.

- (a) We have found that the high permeability of the PDMS lens membrane could lead to loss of fluid by evaporation. To avoid this problem, one should choose other elastomer including different types of PDMS and perhaps more importantly, choose lens fluid of low vapor pressure. We have found different kinds of oil (e.g. lubrication oil) with essentially zero vapor pressure and adequate transparency, index of refraction, and viscosity. One remaining issue for applying such fluid to the lens is its compatibility with the membrane material.
- (b) From the photograph of the lens setup in Fig. 10, it becomes obvious that although the zoom lens is compact and can be made using microfabrication process, the control circuit including the fluidic pumps, tubing, connectors, pressure sensors, valves, etc. takes a much larger space than the lenses themselves. To apply the technology to real systems, we need to fabricate the entire fluidic control circuit in a space comparable to the lenses and using microfabrication techniques. Fortunately, there has been significant progress made in microfluidics and bioMEMS that shed light on the integration of these fluidic devices with the lenses.
- (c) For high performance imaging systems made of solid lenses, all lens surfaces are antireflecting (AR) coated. Experimental and simulation results show that with multiple
 reflections at different lens surfaces, the MTF value could be reduced by as much as
 30%, yielding a significant penalty in image resolution. However, it is nontrivial to
 perform AR coating to the outer and inner surfaces of the membrane since the shape
 of the membrane varies during operation. Conventional ceramic AR coating for solid
 lenses is not applicable here, and new coating techniques have to be developed. Two
 promising techniques for AR coating are polymer coating invented by the MIT group
 and the moss eye patterns. Detailed evaluation of both techniques will be performed
 in collaboration with the MIT group and the RSC group in the BOSS II program.
- (d) Concerning the lens characterization, we have so far only investigated the shape-induced aberration. The effect of chromatic dispersion caused by the membrane and the lens medium has not been studied. To compensate for chromatic dispersion, we believe the most effective way might be using diffractive optics, which typically requires a less number of elements than refractive optics. It would be worthwhile to investigate how diffractive lenses can be incorporated into the refractive fluid lenses using molding process.

5. Publications and Patents

Patent Disclosure

1. "Fluidic Adaptive Eye Correction Lenses and Zoom Lens" D. Y. Zhang and Yu-Hwa Lo, provisional patent was filed in March 30, 2004.

Journal Papers

- 1. D. Y. Zhang, V. Lien, Y. Berdichevsky, J. Choi, and Y. H. Lo, "fluidic adaptive lens with high focal length tunability", Applied Physics Letters, vol. 82, no. 19, 3171-2 (2003)
- Zhang DY. Justis N. Lien V. Berdichevsky Y. Lo YH. "High-performance fluidic adaptive lenses," Applied Optics. 43(4): pp. 783-787, 2004 Feb 1.
- 3. Zhang D, Justis N, Lo YH. "Fluidic Adaptive lens of transformable lens type," Applied Physics Letters, vol. 84, no. 21, May 24, 2004, pp. 4194-4196.
- 4. De-Ying Zhang, Nicole Justis, Yu-Hwa Lo, "Fluidic Zoom-Lens-System-on-a-Chip with Wide Field-of-View Tuning Range," accepted for publication in IEEE Photonics Technology Letters.
- 5. Zhang D, Justis N, Lo YH. "Integrated fluidic adaptive zoom lens", accepted for publication in Optics Letters.

Deleted: Bio-inspired

Deleted: Compact

Refereed Conference Papers

- D. Y. Zhang, N. Justis, and Y. H. Lo, "Versatile lens type tunability in fluidic adaptive lens", IEEE LEOS Conference Proceedings, Tucson, Arizona, October 2003.
- 2. D. Y. Zhang and Y. H. Lo, "Focal length tunable fluidic adaptive lens", CLEO Conference Proceedings, Baltimore, Maryland, 1-5 June 2003.
- 3. D. Y. Zhang, N. Justis, and Y. H. Lo, "Fluidic zoom lens without lens motion", Submitted to 2004 CLEO Conference.
- 4. N. Justis, D. Y. Zhang, Y. Berdichevsky, and Y. H. Lo, "Thin film modification for bio-inspired dynamic microfluidic lenses," MRS Proceedings, Boston, MA, November, 2003.
- 5. D. Y. Zhang, N. Justis, and Y. H. Lo, "Fluidic adaptive zoom lens without lens motion," CLEO 2004 Proceedings, May 16-21, 2004, San Francisco, CA.

6. Technology Transfer

As a result of the BOSS program, we have filed one provisional patent through UCSD. The patent has raised significant interests in imaging and biomedical industry. Agilent, the world's largest supplier of camera modules for cellular phones and PDAs, has expressed interest in licensing the technology. Agilent has offered to develop the technology into a fully manufacturable process for commercialization. On the other hand, AMO, the world's largest company for implanted lenses after cataract removal is prepared to enter a collaboration and licensing agreement with the UCSD group to develop biocompatible adaptive implanted lenses.

Separately, investment communities have shown interest to fund a startup company to commercialize the miniature zoom lens for camera phones and PDAs. The PI also received funding from von Liebig Center of UCSD School of Engineering to commercialize the technology.

Finally, one postdoc and one PhD student sponsored by the BOSS I program will work for government laboratories or private sectors to transfer their knowhow to DoD and industry.

Appendix

I. Abstract

This analysis shows that lens type tunability that allows change of a convex lens into a concave lens can enhance the zoom ratio and reduce the depth of the system.

II. Analysis

We use the following notations in the analysis:

l₁: object distance,

l₂: image plane distance,

d: lens spacing,

f₁: focal distance of the front (object side) lens,

f2: focal distance of the back (image side) lens,

 $\Phi_1 = 1/f_1,$

 $\Phi_2 = 1/f_2$

 $\Phi = 1/f$: power of the zoom lens system,

ZR (zoom ratio).

We have

$$\Phi_2 = 1/l_2 + (1 + \Phi_1 l_1) / (\Phi_1 l_1 d + d - l_1)$$
 (1)

$$\Phi = (-d/l_2) \left\{ \left[\Phi_1 + (d-2 \ l_1)/(2 \ d \ l_1) \right]^2 - (d^2 + 4 \ l_1 \ l_2)/(4d^2 \ l_1^2) \right\} / (\Phi_1 + (d-l_1)/d \ l_1)$$
 (2)

In most situations, we can assume that l_1 approaches infinity, then Eqs. 1,2 can be greatly simplified as

$$\Phi_2 = 1/f_2 = 1/l_2 + 1/(d-f_1)$$
 (3)

$$\Phi = 1/f = (f_1 - d) / (f_1 l_2)$$
 (4)

To form an image on the image plane, $\Phi > 0$ but Φ_1 or Φ_2 does not need to be all positive. This suggests that there is a possibility for extending the zoom ratio with lens type tunability that enables the sign change for either Φ_1 or Φ_2 .

Let us first look at how the power of the zoom lens varies with f_1 by taking $d\Phi/d$ f_1 .

From Eq. (4), we find

$$d\Phi/d f_1 = (f_1)^{-2} (d/l_2)$$
 (5)

Since both d and l_2 are positive, we have proved that Φ increases monotonically with f_1 . According to Eq. (4), $\Phi = 0$ when $f_1 = d$, hence we can conclude that in order to achieve a positive Φ , f_1 has to be greater or equal to d.

Next we try to find Φ_{max} . Since F increases monotonically with f_1 , we obtain Φ_{max} as f_1 approaches infinity. From Eq. 4, we obtain:

$$\Phi_{max} = 1/l_2 \text{ (as } f_1 \text{ approaches infinity)}$$
(6-a)
$$\Phi_{min} = 0 \text{ (when } f_1 = d)$$
(6-b)

However, we have to be careful with the result in Eq. (6), which suggests an infinite zoom ratio. We need to assure that for (6-a) and (6-b), the focal distance of the second lens, f_2 , falls in a reasonable and technologically achievable range. In the following, we will try to find the corresponding value of f_2 for different values of Φ and f_1 .

Region 1: $d+l_2 < f_1 < infinity$

From Eq. (3), we have $1/(d + l_2) < \Phi < 1/l_2$. From Eq. (4), we have infinity $> f_2 > l_2$.

That is

$$\Phi_{\text{max}} = 1/l_2$$
 when $f_1 = \text{infinity}$, $f_2 = l_2$;

$$\Phi_{\min} = 1/(d + l_2)$$
 when $f_1 = d + l_2$, $f_2 = \text{infinity}$.

Assuming that the lens technology allows us to tune f_1 , f_2 so the above focal distances can be achieved, we can achieve a zoom ratio

$$ZR = 1 + d/l_2 \tag{7}$$

Equation (7) shows the maximal possible zoom ratio one can achieve with lenses having a wide tuning range but **without** the lens type conversion capability. In other words, the lenses always have a convex profile.

If the lens technology allows for lens type conversion so its focal distance can become negative, then we can reach the second region with an extended zoom ratio.

Region 2: $d < f_1 < d+l_2$

In the ideal situation, when $f_1 = d$, we have $\Phi = 0$ (from Eq. 4) and $f_2 = 0$ (from Eq. 3). This would have produced a zoom ratio of infinity except that it is impossible to obtain $f_2 = 0$ for any lens design. To calculate the practically achievable zoom ratio, we assume that the minimal magnitude of the negative focal distance for the second lens is f_m .

When the second lens is turned into a negative lens, its focal distance has to satisfy the following condition:

$$f_2 \le f_m \le 0$$

From Eqs. (3,4) and under the condition that $d \le f_1 \le d + l_2$, we can obtain the following relations

$$f_1 > d - l_2 f_m / (l_2 - f_m)$$
 if we require $f_2 < f_m$ (8)

$$\Phi_{\min} = \left[l_2 + d\left(1 - l_2/f_{\text{m}}\right)\right]^{-1} \quad \Phi_{\min} \text{ is obtained when the equality in (8) holds}$$
 (9)

and

$$ZR = \Phi_{max} / \Phi_{min} = 1 + d/l_2 - d/f_m$$
 (10)

Comparing Eq. (10) with Eq. (7), we draw the conclusion that the zoom ratio is increased by an amount of $d/|f_m|$ with the lens type tunability.